

# The Shared Wireless Infostation Model - A New Ad Hoc Networking Paradigm (or Where there is a Whale, there is a Way)\*

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## ABSTRACT

In wireless *ad hoc networks*, capacity can be traded for delay. This tradeoff has been the subject of a number of studies, mainly concentrating on the two extremes: either minimizing the delay or maximizing the capacity. However, in between these extremes, there are schemes that allow instantiations of various degrees of this tradeoff. *Infostations*, which offer geographically intermittent coverage at high speeds, are one such an example. Indeed, through the use of the *Infostation* networking paradigm, the capacity of a mobile network can be increased at the expense of delay. We propose to further extend the *Infostation* concept by integrating it with the ad hoc networking technology. We refer to this networking model as the *Shared Wireless Infostation Model (SWIM)*. SWIM allows additional improvement in the capacity-delay tradeoff through a moderate increase in the storage requirements. To demonstrate how SWIM can be applied to solve a practical problem, we use the example of a biological information acquisition system - radio-tagged whales - as nodes in an ad hoc network. We derive an analytical formula for the distribution of end-to-end delays and calculate the storage requirements. We further extend SWIM by allowing multi-tiered operation; which in our biological information acquisition system could be realized through seabirds acting as mobile data collection nodes.

## Categories and Subject Descriptors

H.1.1 [Models and Principles]: Systems and Information Theory—*Capacity-Delay Tradeoff*; J.4 [Social and Behavioral Sciences]: *Data Acquisition Using Whale Mobility*

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## General Terms

Algorithms, Design, Measurement

## Keywords

Infostation, SWIM, capacity-delay tradeoff, ad hoc, network, animal tag, disease model

## 1. INTRODUCTION

Currently, there are several possible paradigms for wireless mobile communication, for example, cellular, ad hoc, wireless LAN, and *Infostations* [1, 2]. In cellular systems, users enjoy constant connectivity<sup>1</sup> with at least some minimum level of service. Thus, although the delay is small, the capacity is limited as well. In ad hoc networks, the transmission range is significantly smaller than in cellular networks and, thus, the reuse of radio channels can significantly improve the overall network capacity. In mobile ad hoc networks, however, continuous connectivity cannot be guaranteed; nodes can separate from the network leading to network partition. In the *Infostation* model, users can connect to the network in the vicinity of ports (or *Infostations*), which are geographically distributed throughout the area of network coverage. The *Infostation* architecture which was originally proposed by researchers at WINLAB<sup>2</sup>, includes low-power base stations<sup>3</sup> [3]. *Infostations* provide strong radio signal quality to small disjoint geographical areas and, as a result, offer very high rates to users in these areas. However, due to the lack of continuous coverage, this high data rate comes at the expense of providing intermittent connectivity only. In this regard, the above three technologies are instantiations of the capacity-delay tradeoff, which characterizes mobile communications.

Clearly, the choice of technology depends on the application that the network is intended to support. For real-time constant bit rate voice traffic, cellular is a better fit than *Infostations*. However, wireless Internet access may also conceivably be provided with ad hoc technology. Non-delay critical applications, such as certain types of data acquisition systems, may be well suited to the *Infostations* model.

<sup>1</sup>within the limitations of the radio link constraints

<sup>2</sup>Rutgers University

<sup>3</sup>called *Infostations*

More specifically, the *Infostation* network architecture should be used for applications which can tolerate significant delay. Since a node that wishes to transmit data may be located outside the *Infostations*' coverage areas for an extended period of time and must always transmit to an *Infostation* directly, large delays may result. Upon arrival in a coverage zone, the node can transmit at very high bit-rates. Thus, *Infostations* trade connectivity for capacity, by exploiting the mobility of the nodes.

Grossglauser and Tse [4] demonstrate the above capacity-mobility tradeoff by showing that the average long-term throughput per source-destination pair can be kept constant even as the node density increases for a network of mobile nodes. This contrasts with the result shown by Gupta and Kumar [5], which states that as the number of nodes per unit area increases, the throughput of each connection decreases at a rate  $\frac{1}{\sqrt{n}}$ , where  $n$  is the number of nodes. The discrepancy between the two results is explained in the difference of the models used in the two works; [5] assumes fixed nodes, while [4] allows the nodes to move. This mobility increases network capacity. Both [4, 5] assume systems with possibly unbounded delays. In the *Infostations* model, network diversity due to mobility of nodes is exploited by allowing the nodes to transmit at times with larger signal quality to get the improved throughput at the expense of potentially large delays.

In this work, we further extend this capacity-delay tradeoff by proposing a modification to the *Infostations* concept. As the reader may recall, in the *Infostations* model, a user needs to physically travel to the vicinity of an *Infostation* to communicate. We propose allowing information to travel through the network, in addition to the user itself. In other words, the information reaches the *Infostation* by sharing (replicating and diffusing) itself in the network, using the mobile nodes as physical carriers. Due to the nature of the information propagation through the network, we term this networking architecture *Shared Wireless Infostation Model* or *SWIM*. A careful reader will easily recognize that SWIM is a marriage of the *Infostations* concept with the ad hoc networking model. Moreover, propagation of information packets within SWIM resembles the spreading of a disease, with the nodes in the network acting as carriers of an illness, and the sharing of data with other nodes, acting as the transmission of the disease from an infected to a healthy individual. Finally, when a node that carries the packet reaches one of the *Infostations*, the packet is "offloaded" to the network and discarded from the node. In our analogy of the disease epidemic, this is equivalent to an infected carrier reaching a "healing" center, in which the carrier gets rid of the disease. Furthermore, by storing the identity of the packets that were offloaded to an *Infostation*, the carrier will not accept such packets again in the future; i.e., developing an "immunity" to the disease.

What are the benefits of SWIM vis-à-vis the *Infostations* model? Clearly, by allowing the packet to spread throughout the mobile nodes, the delay until one of the replicas reaches an *Infostation* can be significantly reduced. However, this comes at a price; spreading of the packets to other nodes consumes network capacity. Thus, again, we are faced with the capacity-delay tradeoff. We have developed a new way to control this tradeoff by controlling the parameters of the spread; for example, by controlling the probability of packet transmission between two adjacent nodes (analogous to the

probability of infection in our epidemic model), transmission range of each node (analogous to the infection distance in our epidemic model), or the number and distribution of the *Infostations* (analogous to the number and location of the hospitals in our epidemic model). SWIM also allows us to study another aspect of the capacity-delay tradeoff, namely the amount of storage required to realize a particular instance of this tradeoff.

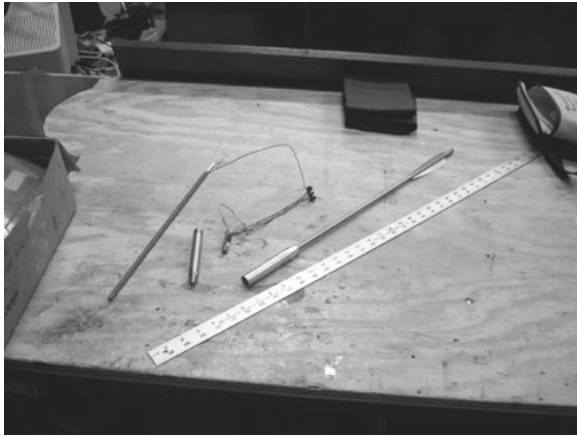
In this paper, we present the study of the SWIM concept through evaluation of an example application: the biological information acquisition system. We model the system analytically and provide numerical evaluations of certain parameters. This allows us to learn about the trends in the system performance as system parameters vary. This also permits us closer examination of the capacity-delay tradeoff. Biological systems fall into the category of unbounded delay discussed earlier. We can assume that since animals have been behaving similarly for years, old data may be as valuable as recent data and we say that data is not time-critical on any time-scale. Due to space limitations, we omit from this paper more extensive modeling and analysis. In particular, we do not present here how the vast knowledge in epidemic modeling could be leveraged from in studying SWIM. Those results will be presented in a future publication.

In the next section, we examine the whale data acquisition problem and consider some of the biological incentive for tracking and studying whales. Next, we propose a particular method of information sharing and study the suitability of the SWIM architecture, so that rather than having each whale transmit directly to an *Infostation*, whales that come in sufficiently close contact with each other may share information. Then, any of these whales that come in the vicinity of a SWIM station could upload all the stored data.

## 2. A BIOLOGICAL INFORMATION ACQUISITION SYSTEM

Due to the ever-decreasing whale population, biologists wish to collect data about these creatures to help to preserve them [6]. In particular, whales and other marine mammals are currently of concern in relation to several marine environmental issues. Underwater noise pollution has been shown to affect these creatures severely. For example, upon hearing noise from underwater tests, *beluga whales* will often flee the location at full speed for 2-3 days and not return to the site for weeks. *Beaked whales* have been stranded in association with naval exercises in many circumstances. The tagging and tracking of these animals can provide more data about their natural behavior and response to human disturbances, such as heart rate, body temperature, speed and acceleration, dive depth and shape.

Currently, one of the primary methods of collecting data from free-ranging animals is radio tagging. Radio frequency devices are implanted or attached to animals, and transceivers measure and collect data concerning the animal and its environment. This data is collected in a continuous manner, then partitioned into discrete packets, and stored in memory with packet identifiers. An example of such a tag is shown in Figure 1. Currently, radio tags are most often used with Argos LEO satellite service [7]. Data collected on the whales is offloaded to a satellite, if the whale is surfacing while the satellite orbits overhead. The SWIM model offers a lower



**Figure 1: Whale tag prototype, to be delivered using a crossbow**

cost alternative to the satellite system, where packets that have been stored on the whales are transmitted directly to the SWIM stations.

Most whales follow consistent migration and surfacing patterns. Different types of whales have known typical dive times; times during which they remain underwater without surfacing. After several dives, the whales socialize and feed near the surface of the water for minutes or hours. Some whales are known to return to the same feeding grounds at regular intervals. These grounds offer excellent locations for the placement of offloading stations (i.e., the SWIM stations), allowing for unpredictable movement of the whales, then their return for upload at the known locations. As well, these grounds attract many different whales and present an excellent opportunity for information sharing, as we will discuss later.

Since the whales spend much of their time underwater, where RF signal transmission is not practical, the brief time they surface may not be sufficient to offload extensive amounts of information at the very low data rate of the satellite tags. As a consequence, we propose to implement a new biological information acquisition system through the use of the SWIM networking architecture, replacing the rather costly and low data rate satellite collection. A network of close-range, high-bandwidth stations, previously described as the SWIM architecture, in addition to low-cost and low-power tags, is precisely a solution that could allow the tags to upload their information quickly and efficiently.

Specifically, in our system, whales are equipped with a larger amount of memory than in today’s “direct offloading” systems. Data that is collected on a particular whale is stored locally. However, as a whale comes in close proximity to another whale, the stored information may be transmitted<sup>4</sup> and stored in the other whale’s memory as well. As the whales migrate throughout the system, a whale that comes in close contact with one of the SWIM stations, offloads all the data in its memory (whether its own data or data from other whales) onto the SWIM station. Thus, as the whales feed and socialize, the devices upload the packets of data to the appropriately placed SWIM stations at high bit-rate. After offloading its stored information, the whale’s memory is then cleared.

<sup>4</sup>with some “infection probability”

Of course, implementation of such a system requires to address a number of critical design choices, such as the probability of transmitting the information from one whale to another, the transmission range of the whales and the SWIM stations, the amount of storage which should be placed on the whales, the time a packet should be deleted from a whale tag, etc. Additionally, there are many alternative system designs. For example, consider the problem of deletion of the information from the whale storage. One possible design alternative is to estimate the probability, as a function of time, that a piece of information will be offloaded to a SWIM station. Then, by putting a “timestamp” on each piece of information, the information will be deleted from the memory of every whale when the timestamp expires. Another alternative could be to use “invalidate” packets, so that whenever a piece of information is offloaded, the invalidate packet is injected into the whales and spread throughout the system, erasing the information from any whale that the invalidate packet visits. In this work, we employ the first choice, leaving the examination of the second and other strategies for future works.

In particular, to estimate the time by which a piece of information can be deleted from the system, the number of copies of the information in the system is modeled as a discrete Markov chain, following the model of an infectious disease [8]. From this Markov chain, the time that a packet is present in the system is found, and the termination criteria is determined based on the desired probabilities that the packets are received at the stations. Our results show how to choose expiration times for the packets for a desired reliability level of delivery. This is equivalent to imposing a probabilistic end-to-end delay constraint in typical ad hoc networks. Given these expiration times, we then calculate the required storage capacities for different design scenarios and mobility models. Trends for a variety of design parameters are also investigated.

### 3. OUR MODELS

#### 3.1 Network model

As the whale surfaces and exposes its RF antenna, it may transmit to a close-by SWIM station, usually one within the line of sight. Typically, the SWIM stations will be placed on buoys, floating in the water. Naturally, one would distribute several stationary SWIM stations within the transmission range of the known whale paths. After receiving and storing the information from the whales, the SWIM stations would transmit the information to shore, either by coordination with other SWIM stations, for example by creating another ad hoc network of SWIM stations, or directly to a satellite, whenever the next satellite passes overhead. Note that we leave open the question of how the information is conveyed from the SWIM stations to the terrestrial network, as the design of the network of whale tags is, to a large degree, independent from the design of the inter SWIM station connectivity. We do, however, consider the effect of mobile SWIM stations in a later section of this paper, assuming that the SWIM stations have infinite capacity. We intend to address in more detail the design of the network of the SWIM stations in our future work.

Moving information from a whale tag to a SWIM station may be time-consuming. This time is reduced by placing several SWIM stations along the whales’ path, since a whale

could surface near any of these SWIM stations and offload its information. Use of the information sharing model further improves the chance of getting the information ashore quickly. However, using the SWIM model implies that a whale tag would store not only its own data, but also information from other whales that is transmitted whenever one whale is within transmission range of another. In this way, only one whale would need to surface near a SWIM station for a piece of information to be transmitted to shore. Finally, we can additionally improve this delay by allowing the SWIM stations to move themselves, such as, for example, attaching the SWIM stations to seabirds; this extension will be discussed later in the paper.

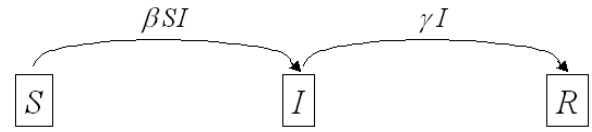
Though the whales are collecting data in a continuous manner, for the purpose of transmission of the information, it is more efficient to divide the data into packets. For example, data collected during one day could be considered one packet, separate from a packet collected on a different day. Whales cannot transmit partial packets, though they may transmit any number of whole packets during a single surfacing event of a whale. Correctly sizing a packet is also important, as overly large packets would reduce the chance of the whole packet being received during a single surfacing event, while overly small packets would lead to high overhead. Each packet carries a distinct identifier, so that a whale or a SWIM station will reject packets which have already been received in the past. Finally, each packet is stamped with a time stamp indicating its creation time and the required Time-To-Live. When the Time-To-Live of a packet expires, it is discarded from the whales' tags.

This Time-To-Live field eliminates the need for clock synchronization between the whales. The "expiration time" is known a priori, calculated by using system parameters as described later. When the whale shares a packet with another whale, the Time-To-Live of the transmitted packet is reduced by the duration that the source whales carried the packet, so the Time-To-Live field carries the remaining time. Thus, all timing is performed using local clocks and no synchronization between whales is needed.

Delay experienced in the network varies considerably depending on the species of whale which carries the tag. One might expect nearly daily surfacings near SWIM stations for whales off the coast of the Hawaiian islands, leading to delays on the order of hours. Whereas, migratory whales may only visit known feeding grounds twice a year, so delays may be on the order of months.

In this paper, we address neither the design of the radio transmission system, nor the design of the MAC layer protocol for transmission between a tag and a SWIM station or between two whale tags. For the purpose of this work, any protocol with reasonably good reliability and some form of acknowledgement of a correctly received packet will suffice.

As the devices implanted in the whales perform measurements, the collected information is divided into separate packets, which are stored in the memory of the whale tag. Anytime one whale comes within transmission range of another whale, the tags on the two whales share information by exchanging the stored information packets. Anytime a whale comes within transmission range of a SWIM station, the information from the whale tag is transmitted to the station and erased from the tag. Furthermore, when the timestamp on the packet expires, the packet is discarded from the tag's memory. Due to the information sharing, the



**Figure 2: Markov chain model of an infectious disease with susceptible, infected, and recovered states**

storage requirement of a whale tag increases. To evaluate this extra storage requirement, we use Markov Chains for modeling of infectious diseases to determine the packet termination criteria. More specifically, given the probability level, we develop a formula for the necessary Time-To-Live of a packet, so one can be confident that with this probability level, the packet will be offloaded onto one of the SWIM stations in the system.

### 3.2 Analytical model

The propagation of each packet of data information generated by a whale can be modeled as spread of one infectious disease. The modeling of diseases in fixed networks have been studied in the past [9], and this model is used in a similar manner in this case. Using the framework from an infectious disease model [8], a whale is "infected" if it has the data packet stored in its memory. The whale is "susceptible" (to infection) if it does not yet have the packet stored in memory, but could potentially acquire the packet from another whale. The whale is "recovered" (healed from the disease) if it has offloaded its information to a buoy (a SWIM station). Note that through the use of the packets' identifications, a packet will be stored only once in each tag (one cannot be infected multiple times with the same disease). Furthermore, by storing the identifications of the previously received packets, a whale may become "immune" to receiving the same packet again.

Therefore, as shown in Figure 2, the susceptible state  $S(t)$  represents the number of whales in the system which are "susceptible," infected state  $I(t)$  represents the number of "infected" whales, and  $R(t)$  represents the "recovered" whales.  $\beta$  represents the contact rate of the whales. Suppose there are  $N$  whales in the system, then a whale contacts  $\beta(N-1)$  other whales per unit time, of which  $\frac{S}{(N-1)}$  do not yet have the disease. Therefore, the transition rate from state  $S$  to state  $I$  becomes

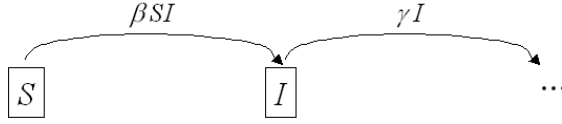
$$\begin{aligned} \text{total infection rate} &= (\# \text{infected})(\text{contact rate})(\# \text{ susceptible whales}) \\ &= I[\beta(N-1)]\left[\frac{S}{(N-1)}\right] = \beta SI \end{aligned}$$

The recovery rate is labeled as  $\gamma$  – it is the contact rate of a whale with a SWIM station. Then

$$\begin{aligned} \text{total recovery rate} &= (\text{whale-buoy contact rate})(\# \text{ infected whales}) \\ &= \gamma I \end{aligned}$$

Recall that if there are multiple buoys, then  $\gamma$  represents the contact rate per buoy; e.g.,  $\gamma$  will double if the number of buoys is doubled.

In this paper, one objective will be to find a distribution of values  $T$ , where  $T$  is the time from the packet generation until the first time the packet is offloaded to a buoy; that is the time until any one whale moves to state  $R$ . As a result, the entire number of whales  $N$  will be contained in the two states  $I$  and  $S$ . Since we assumed that different packets in



**Figure 3: Markov chain model of an infectious disease without 'recovered' state, since a transition to the recovered state represents the end of the simulation**

the system propagate independently, we consider only one packet at a time. Thus, at time  $t = 0$  only one whale is infected with the packet. Therefore, the initial conditions for the system are:

$$S(0) = N - 1, \quad I(0) = 1, \quad S + I = N, \\ R(t) = 0 \text{ for } t < T \text{ and } R(T) = 1$$

First we find  $I(t)$ , the number of infected whales as a function of time. Then, the cumulative probability distribution for  $T$  is derived. Once the cumulative distribution  $F(T)$  is known, it is possible to choose a desired probability level, so that at some time  $\tau$ ,  $F(\tau)$  equals this probability level. In other words, by time  $\tau$ , one can be confident with the selected probability level,  $F(\tau)$ , that the packet has reached at least one of the buoys in the system.

### 3.3 Solving for cumulative distribution $F(T)$

We are interested in the transient solution to the Markov Chain in Figure 3, with the following first-order differential equation:

$$\frac{dS}{dt} = -\beta SI \\ \frac{dI}{dt} = \beta SI = \beta(N - I)I = \beta NI - \beta I^2$$

This differential equation is separable and can be solved with the initial condition  $I(0) = 1$  to give the solution

$$I(t) = \frac{N}{1 + e^{-\beta N t}(N - 1)},$$

which can then be used to find the cumulative distribution function

$$F(T) = 1 - K \left( \frac{N - 1}{e^{\beta N T} + N - 1} \right)^{\frac{\gamma}{\beta}}. \quad (1)$$

The reader is referred to the Appendix for more details about the solution of (1).

### 3.4 Random midway mobility model

Solution of the above analytical model depends on two parameters,  $\beta$  and  $\gamma$ . In particular, these two parameters are strongly affected by the physical set up of the system, such as the mobility model of the whales, the number and the placement of the buoys, the infection radius of the buoys and the whales, etc. We first consider the choice of a simple mobility model, the random midway mobility model.

In the random midway mobility model, the whales swim in straight lines for a fixed small time interval,  $\delta t$ , with a randomly chosen velocity from  $[v_{min}, v_{max}]$  and in a random direction distributed uniformly between 0 and  $2\pi$ . At the beginning of each time interval, a new velocity and a new direction are chosen.

Unfortunately, even for this relatively simple mobility model, estimation of the two system parameters, the  $\beta$  and the  $\gamma$ ,

is not trivial. Thus, we resorted to simulation for evaluation of these two parameters. In particular, we have developed an event-driven simulation, which models the whales' movement and computes the contact rates among the whales and between the whales and the buoys, using the random midway mobility model above, with  $v_{min} = 0[m/s]$  and  $v_{max} = 6[m/s]$ . The whales swim along a rectangular region with edges that wrap around, creating a toroidal surface.

Parameters of the simulation include: the infection radius for the whales and for the buoys, the number of whales, the number of buoys, and the size of the rectangular region. One whale generates one packet at the beginning of the simulation. At every iteration, the distances between each whale and all other whales and buoys are calculated. If a whale carrying a packet is within the infection range of another whale, then the packet is replicated at the other whale. If any whale carrying the packet is within infection range of a buoy, then the simulation is stopped, recording the time,  $T$ , from the creation of the packet until the termination of the simulation. The number of whales infected at that time equals the number of the packet replications that might be expected in the system. The simulation is run multiple times, and the data is compiled, representing the cumulative distribution function,  $F(T)$ . This  $F(T)$  obtained through simulation will be then compared with the analytical solution obtained in the previous section. However, to do so, we need first to evaluate the two parameters  $\beta$  and  $\gamma$  - we proceed with this task in the next section.

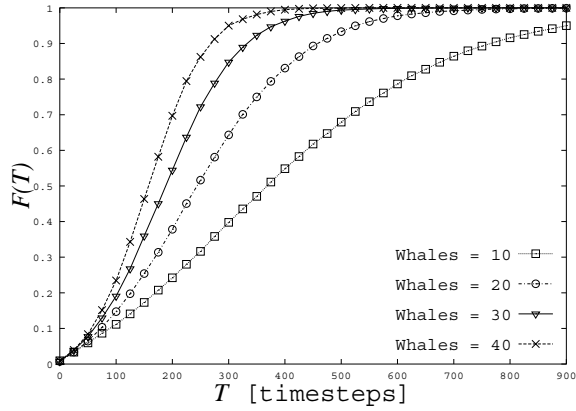
The simple random midway mobility model was extended to a more sophisticated model involving migration and grouping tendencies of the whales toward each other and toward feeding grounds. A further extension of the model involves motion of the SWIM stations (e.g., replacing buoys in the paper). These extensions are discussed later in the paper. First, we examine the results of the random midway mobility model.

### 3.5 Comparison of analysis and simulation

To proceed with our evaluation of the biological information acquisition system, we need to estimate the parameters  $\beta$  and  $\gamma$  using the simulation (previously used to obtain  $F(T)$ ). The contact rate  $\beta$  is calculated by recording the times between infections at each value of  $I$  for many simulations. For example, the time interval,  $\Delta t$ , from the time the system enters the state  $I = 2$  until it moves to the state  $I = 3$  is recorded and averaged for many simulations. Since the rate from state  $S$  to state  $I$  is  $\beta SI$ , then  $\beta \approx \frac{1}{ST\Delta t}$ . These values for  $\beta$  are then averaged over the different states  $I$  to give an overall  $\beta$  for the system.

Recall that  $\gamma$  quantifies the rate of infection of buoys per whale, since the rate of whales leaving state  $I$  is  $\gamma I$ . Since  $\gamma$  is independent of  $I$ , running the simulation, while counting the number of buoys that *would be* infected during some long time period without actually infecting these buoys, allows for the calculation of the contact rate while keeping  $I$  constant. This rate equals  $\gamma I$ , and thus  $\gamma = \frac{\text{rate of infection of buoys}}{I}$  may be calculated.

Equipped with the values of the parameters  $\beta$  and  $\gamma$ , we are now able to compare the theoretical  $F(T)$  distribution with the  $F(T)$  obtained from the simulation. For the comparison, we utilize the  $\chi^2$  statistical test[10], which is used to verify the hypothesis that a sampled process is likely to have arisen from a given distribution. In our case 10 bins



**Figure 4: Cumulative distributions of  $T$ , the time from packet creation until offloading, for different numbers of whales in the system**

were used in the  $\chi^2$  test, so if the  $\chi^2$  statistic is lower than 1.152 - the critical value of a  $\chi^2$  distribution with 9 degrees of freedom - then the distributions are said to agree with 99.9% certainty. Data compiled in the Appendix shows the excellent agreement between the simulation and the theory for many different values of the parameters using this uniform random midway mobility model. From this table, we can see the direct linear dependence of  $\gamma$  on the number of buoys; e.g., as we would expect, twice as many buoys leads to a  $\gamma$  value which is twice as large. Also, the  $\gamma$  values are independent of the number of whales, as each infected whale has the same opportunity to upload its packet to a buoy, regardless of the number of whales around it.<sup>5</sup>

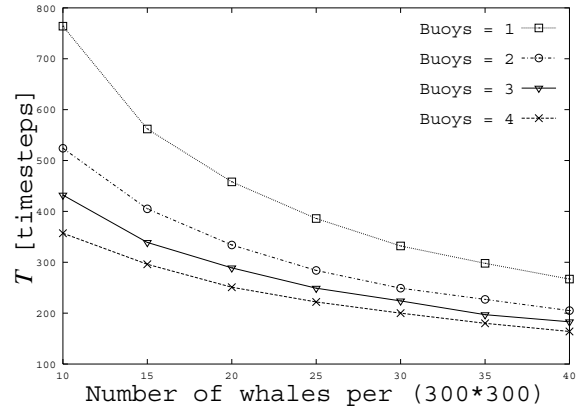
As expected, the table shows the  $\beta$  values are independent of the numbers of buoys. The  $\beta$  values are nearly constant, though they do decrease slightly with an increase in number of whales. This is due to the occasional overlap of transmission regions of the whales as they swim on the flat torus and implies that  $\beta$  is somewhat dependent on  $I$ . Our model approximates the system by assuming the overlap of these regions is negligible. Though this approximation might not be good for very dense networks (a very large number of whales present in a small area), the case that we consider here is sufficiently sparse, so that the overlap phenomenon is, indeed, negligible. The match of the analytical and simulation results confirms this claim.

### 3.6 Parameters and Quantities of Interest

A result of the simulation is the cumulative distribution function,  $F(T)$ , and is depicted in Figure 4 for different number of whales in the system,  $N = 10, 20, 30, 40$ . From the  $F(T)$  curves, one can pick a threshold probability  $P_{\text{thresh}}$  to find the time at which with probability  $P_{\text{thresh}}$ , the packet will be offloaded to one of the buoys; i.e.,  $T = F^{-1}(P_{\text{thresh}})$ . For example, if  $P_{\text{thresh}} = 0.9$  then the required  $T$  is shown in Figure 5.

We have examined the effect of the parameters in the mobility model on  $F(T)$  and the number of infected whales at time  $T$ . Due to space limitations, we omit the detailed description of those outcomes, only stating some selected results. Changing the speeds of the whales amounts to assum-

<sup>5</sup>The overall rate out of state  $I$  is  $\gamma I$ , which increases as we increase  $I$ , but  $\gamma$  is the contact rate per whale to any buoy and this is constant.



**Figure 5: Average delay  $T$  until the packet is offloaded with probability 0.9**

ing a different number of seconds per timestep. This does not change the curves, since we plot in units of timesteps rather than minutes or hours. The placement of buoys has some effect on the  $F(T)$  distribution, but this effect is limited and is exhibited in a more interesting manner in the mobility model discussed in the following section.

## 4. GROUPING/FEEDING MOBILITY

We have experimented with more sophisticated whale mobility models that more realistically capture the physical whale behavior by, for example, incorporating feeding grounds. In general, these feeding grounds are centers of attraction for the whales, where the whales typically swim slower or even stop at times. In this enhanced model, three issues govern the direction of the whales' positions at any time: migration, grouping of whales, and direction of the nearest feeding ground. Females tend to group together with other females, while grown males tend to be more solitary and group with females but not with other males. Many papers in biological journals support these dominant factors of the whale mobility model [11, 12, 13].

In the simulation results presented here, the speed of a whale is randomly chosen between  $v_{\text{min}} = 2[m/s]$  and  $v_{\text{max}} = 3[m/s]$  for whales outside the feeding areas (though males travel twice as fast 20% of the time) and between  $v_{\text{min}} = 0[m/s]$  and  $v_{\text{max}} = 1[m/s]$  when within a feeding area. Any whale may choose to stop inside a feeding area, however only males take short breaks when traveling outside the feeding areas, since they travel faster and, typically, get tired more quickly. Direction for the whales is determined by a weighted vector sum of the directions of migration, of the direction to the nearest female, and of the direction to the nearest feeding area.

It is worthwhile to emphasize that our analytical solution, as expressed by equation 1, is still valid. The changes to the physical parameters are all captured in the values of  $\beta$  and  $\gamma$ . Thus, to evaluate the system through the use of our analytical model and under a different mobility pattern, we only need to recompute  $\beta$  and  $\gamma$ .

### 4.1 Finding the direction vector

As stated earlier, the overall direction vector for each whale is calculated by incorporating these individual mobility elements. This is achieved by taking a weighted average of the migration pattern of the whale, the location of the

closest female in the vicinity of the whale's position, and the direction of the closest feeding area. We first discuss each mobility element separately.

A migration direction is defined for each whale. This direction reflects the general travel goal of the whale, although the migration direction is modulated by a random whale behavior. The migration direction is not an absolute direction, but rather an angle (in radians) that is the mean of a Gaussian distribution representing the migration angle [14]. Every timestep,  $\delta t$ , the whale generates another Gaussian with a mean of the migration direction and some standard deviation. Increasing the standard deviation introduces more randomness into the mobility pattern of the whale. Migration vectors are always assumed to have weight 1.

If a given whale is within sensing range of other whales, it will set its grouping vector in the direction of the closest female. Whales which are already grouped (within half of the transmission range of a female) need not set their grouping vector. It is assumed that the mobility of each whale in the group would then be governed by the same migration and food, and they would tend to stay grouped together, swimming in parallel.

The grouping weight is another Gaussian variable, which, in our simulation, has the mean of 0.8 and the standard deviation 0.5. To help the whales break away from groups, there is also a *maxGroupingTime* variable, set to 3 in our simulation. If the whales have grouped for longer than the *maxGroupingTime* then the grouping weight is changed to 0.0 for the next iteration. These values offer a model which appears to empirically resemble the whales mobility patterns, where some of the whales group and others break off from the groups.

The whales will swim toward feeding/socializing areas if they are hungry. These areas are modeled as circles of radius on the order of 20 to 30 km at the surface of the water and with specified center locations. The feeding areas are separated by a distance, which we have assumed to be about 200 km in our simulation. The distance from a whale to the closest point of a feeding area is calculated as

$$[(\text{whale position} - \text{center of feeding area}) - (\text{radius of area})],$$

and the feeding area contribution of the overall whale's direction vector is a unit vector pointing in the direction of the closest feeding area.

The *hungry* variable of a whale measures the whale's feeling of hunger. If the whale is swimming outside the feeding area, the *hungry* variable is increased, while it is decreased inside a feeding area. A whale will be more attracted to a feeding area, the hungrier it is. The contribution of the attraction to food is expressed as a weighting factor, modeled as Gaussian variable with mean  $0.4 * \text{hungry}$  and standard deviation  $0.3 * \text{hungry}$ .

A motion consistency factor must also be considered, since it is quite unlikely for a whale to completely change direction at each timestep. The consistency factor takes values between 0 and 1. The old direction, multiplied by the consistency factor, is added to the new direction, multiplied by  $(1 - \text{consistency factor})$  and the result is again normalized to a unit vector. This, effectively, implements a low-pass filter for the overall mobility direction vector.

With the resultant direction unit vector and speed, the

whale locations are updated at every  $\delta$  time interval:

$$\text{new location} = \text{old location} + (\text{speed}) * (\text{direction unit vector}).$$

## 4.2 Results for Grouping/Feeding Mobility

We have experimented with the extended mobility model and obtained results that indicate an agreement between the simulation and the theory for different values of the parameters. These are shown in the Appendix table.

Using this extended mobility model, there are many choices of the parameters to examine. As with the random midway mobility model, we study trends by varying the density of whales, density of buoys, the infection radii of the whales and of the buoys, as well as the speed of the whales and the placement of buoys. A more subtle parameter is the percentage of female whales. Reducing the percentage of female whales amounts to less attraction between whales, since the males are attracted to the females, but not to other males, and the females also group with females. The "sensing region" parameter is the radius of the circle where a whale can "sense" a nearby feeding ground. If the distance between the whale and any feeding ground is larger than this sensing radius, then the whale is not attracted to the feeding ground. Decreasing the consistency factor induces faster reaction times to nearby food or grouping with nearby whales. Low consistencies quickly degenerate the system, so that whales group very closely to each other and feeding grounds resulting in larger delays.

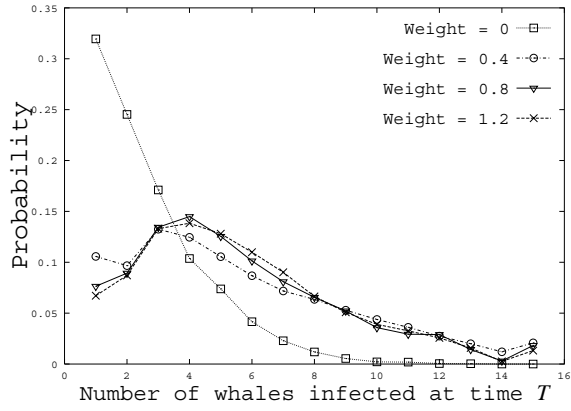
Due to space limitation, we present here only selected results. In particular, we show the effects of the tendency toward other whales, the arrangements of buoys, and the probabilities of successful transmission on the simulation results. We consider how the average time until packet offloading and the number of whales infected at  $T$  are affected by varying grouping weights and SWIM station positions.

Figures 6 and 7 consider different grouping weights for a system with buoys at maximum separation on the toroidal surface. To examine grouping weights without feeding effects, we set feeding weights to zero. Since there is no tendency toward the feeding grounds, there is no advantage to placing buoys in any particular location, and we place the buoys uniformly throughout the area for these tests.

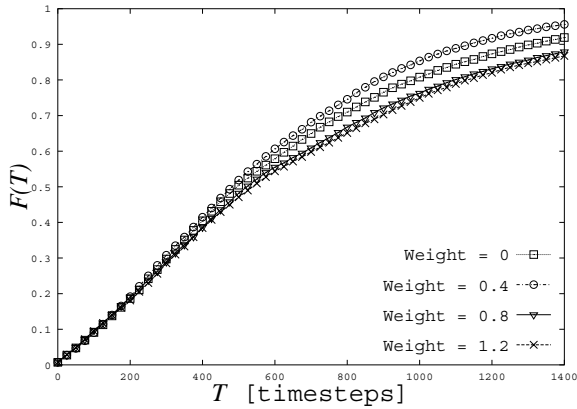
Figure 6 shows the distribution of the number of infected whales (out of the total  $N = 15$ ) at the offload time  $T$ . We observe that, as the grouping weight decreases, fewer whales are infected at the offloading time. Thus, for more solitary whale behavior and all else equal, there are fewer copies of a packet present in the network.

Figure 7 shows that increasing the grouping weight first increases the probability of packet offloading at time  $T$ , shortening the average packet offloading time. This is expected, since higher grouping increases the number of copies of the packet in the system. However, increasing the grouping weight too much actually decreases the probabilities of packet offloading. More grouping of whales results in a smaller spread in overall whale location, so there is less opportunity for offloading of packets and the delays increase. For these reasons, we can postulate that for any set of parameters, there is some optimum grouping weight, such that  $F(T)$  increases most quickly with  $T$ . For the case shown in the Figure 7, this weight is near 0.4.

Feeding weights variables determine the magnitude of whales'

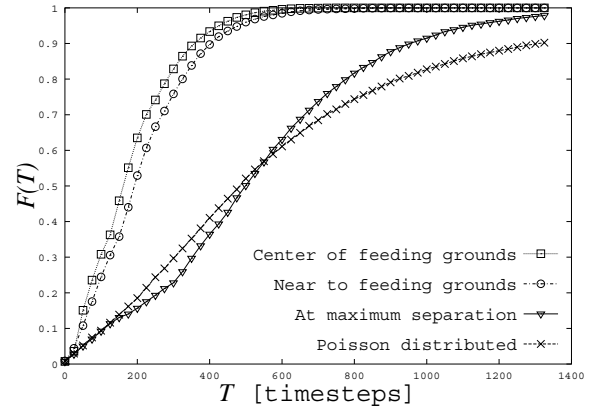


**Figure 6: Comparison of distributions of copies of the packet in the system at  $T$  for different grouping weights**



**Figure 7: Cumulative distributions of  $T$  which show the effect of varying the tendency of the whales toward other females**

tendencies toward feeding grounds. Varying non-zero feeding weights results in little difference in the  $F(T)$  curves obtained. Varying the number of feeding grounds shows more effect. As one might expect, having more feeding grounds increases the delay, because there is a larger chance that whales may be attracted to locations without SWIM stations. However, if the stations are placed in the middle of these feeding grounds, then the whales are more likely to reach the vicinity of an SWIM station. For this reason, the most significant differences are shown when the buoys are placed inside the feeding grounds, as opposed to outside the feeding grounds. Figure 8 assumes 1 buoy in the system and 15 whales in each of the four arrangements of the buoys: buoys at the center of feeding grounds, buoys near feeding grounds, buoys uniformly distributed away from the feeding grounds, and buoys distributed randomly with Poisson distribution throughout the area. Figure 8 shows that the situation with SWIM stations at the center of the feeding grounds leads to the shortest offloading times, since whales are attracted to feeding grounds and are likely to be within range of a SWIM station by approaching a feeding ground from any direction. The situation with SWIM stations near (rather than at the center of) the feeding grounds is most likely to arise in practice, such as placing the stations on the shore near the feeding grounds. Fortunately, this situation also shows similarly short offloading times, because the



**Figure 8: Effect of different buoy arrangements on the cumulative distribution of  $T$**

attraction of the whales to the feeding grounds continues to place the whales near the SWIM stations. The situation with stations at maximum separation places one buoy outside the feeding region, then places all other buoys at maximum Euclidean distance from each other. This results in a curve which suggests curious behavior: first increasing slowly, then more quickly. This phenomenon is a result of the relationship between the position of the feeding grounds and the position of the buoys, which gives preference to certain  $T$  values. In the Poisson distributed case, the attraction of the whales to the feeding grounds offers no advantage, so the  $F(T)$  curve increases steadily.

Figure 9 shows that very few whales are infected with the packet, when the stations are at the center of the feeding grounds. This is due quick offloading times. Since whales are attracted to the SWIM stations, they are likely to offload the packet before having a chance to share the information with many other whales. For the same reason, the SWIM stations near the centers of the feeding grounds show the same trend. The maximum separation case shows that frequently almost all whales become infected with the packet before it is offloaded. This is due to the fact that the whales are attracted to the feeding grounds which are relatively far from the SWIM stations, so there is much more opportunity to share the information packets. The Poisson case shows a more even distribution of the number of whales which carry the packets, since the position of the SWIM stations is completely independent from feeding grounds, which represent locations of attraction for the whales.

To save capacity in the network the user might wish to control the probability with which a packet is shared between nodes. We can say that there is probability  $p$  of successful transmission given an opportunity to exchange data, and analyze  $F(T)$  curves for different  $p$  values. Figure 10 shows that for the mobility pattern using grouping weight 0.8, feeding weight 0.4, 15 whales, and 1 buoy placed near the feeding ground, any success probability higher than 0.4 has an  $F(T)$  curve nearly identical to the  $p = 1$  case.

## 5. MULTITIER MOBILITY

Up to this point, we have assumed that the collection points are fixed in their locations. Another possible model for the biological information acquisition system is to consider mobile collection points as well as mobile nodes – for example the case where the SWIM stations are mounted on



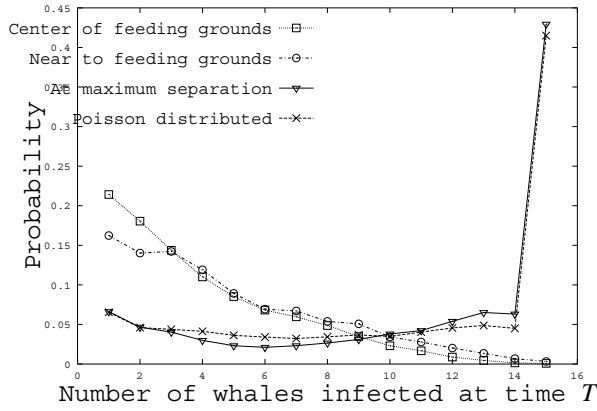


Figure 9: Effect of different buoy arrangements on the number of copies in the system

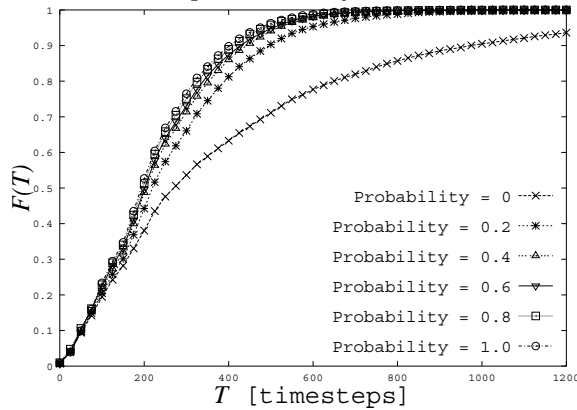


Figure 10: Cumulative distributions of  $T$ , varying probabilities  $p$  of successful packet transmission

seabirds, that glide above the ocean along the turbulent air above the waves. Using the same system parameters as before (i.e., the same number of whales and buoys), the graph in Figure 11 shows that considerably increasing the speed of the buoys has a positive effect on the packet offload time. However, at a SWIM station speed of 3, higher delays are observed in the system. Since the whales travel at speeds close to 3, the speed 3 stations are less likely to pass through a group of whales; it is likely that the distance between the stations and the whales would remain constant. The higher speeds of the buoys allow the mobile stations to pass through groups of whales more often, and to stay close to the groups for shorter periods each time. This higher frequency of visiting allows the packets to be offloaded more often and at more regular intervals.

Though varying the grouping weights has some small effect on the system for stationary SWIM stations, there is no effect observed by varying the grouping weights for mobile SWIM stations either on the  $F(T)$  or on the distribution of the number of packets in the system at time  $T$ .

## 5.1 Designing a system

The  $F(T)$  curves and the numbers of infected whales for each packet allow us to design a biological information acquisition system. In designing the system, since there is no interaction among the packets in the system, each packet is assumed to propagate independently of any other packet. This assumption is reasonable, as long as there is no priority

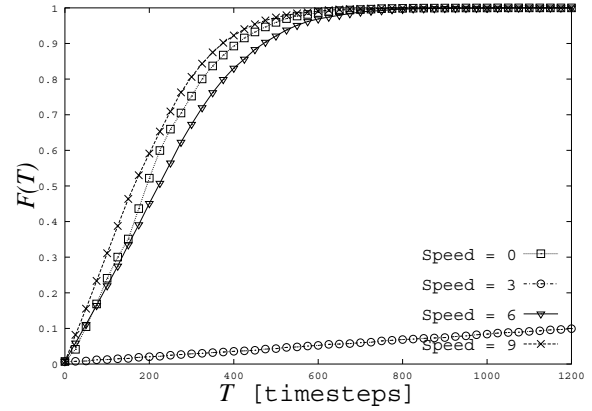


Figure 11: Cumulative distribution curves with varying speeds of SWIM stations

assigned to the packets.

Deep diving whales surface from a dive roughly once every 40 minutes, so a packet containing information from 10 dives is generated every 400 minutes. Each timestep in the simulation represents 1000 seconds, if each spatial unit represents 1 km. Therefore, for each whale, the rate of generation of packets is  $\frac{1000 \text{ [sec/timestep]}}{400 \text{ [min/packet]} * 60 \text{ [sec/min]}} = \frac{1}{24} \text{ [packets/timestep]}$ . Each packet stores the information for 10 dives in 330 bytes. This memory accounts for packet ID, position, coefficients representing the dive, and parameters about the water column.

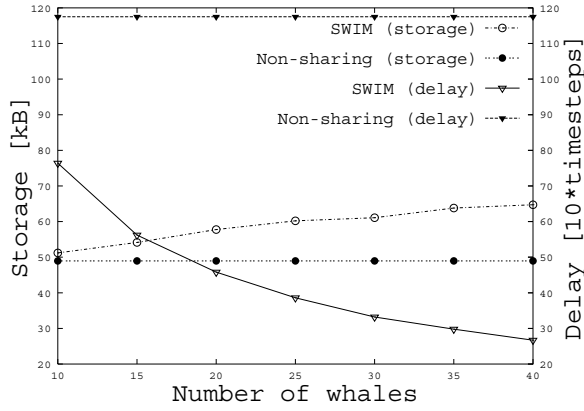
The designer of the system first specifies a threshold probability  $P_{\text{thresh}}$  with which the packets should be offloaded to a station. Suppose the designer chooses probability of 0.9 that any given packet from a whale will eventually reach a station. Then, an  $F(T)$  curve is plotted to find the appropriate value of  $T$ . The Time-To-Live field in the packet is set to this value of  $T$ , to allow packets to “expire” and to be removed from the network.

Suppose 15 adult whales are tagged and swim in an area roughly 22500 km<sup>2</sup>, where 1 buoy is placed for data collection near a known feeding ground. The transmitting range of the radio tags is 3.25km and reception range of the stations is 7.5 km. This can be modeled as system with  $N = 15$  whales, 1 buoy, food weight of approximately 0.4, and the buoy position near, but not in the center of the feeding ground. Utilizing the appropriate  $F(T)$  curve from Figure 8, we see that  $F^{-1}(0.9) \approx 562$ . The “expiration time” of the packets is therefore 562 timesteps.

Using Little’s formula with generation rate  $\lambda = \frac{1}{24}$  timesteps per packet per whale, the expected number of all the packet replicas in the system is:

$$\begin{aligned} EP &= (\text{number of whales})\lambda ET \\ &= 15 \frac{1}{24} \text{ [packets/timestep]} (562 \text{ [timesteps]}) \\ &= 351.25 \text{ [packets]} \end{aligned}$$

An estimate of the number of replicas of each packet in the system,  $EI$ , is the expected number of whales infected by a packet at time  $T$ . It can be shown from simulation that  $EI = 7.1759$  in this case. This number assists in the calculation of a global storage requirement for the radio devices,



**Figure 12: Necessary storage requirements and expected delays using SWIM vs. the non-sharing model**

which is calculated as follows:

$$\begin{aligned}
 &\text{storage requirement} \\
 &= (\text{duplicates})(\text{different packets})(\text{bytes/packet}) \\
 &= EI * EP * (330 \text{ bytes/packet}) \\
 &= (7.1759) * (351.25) * (330 \text{ bytes/packet}) \\
 &= 831777 \text{ bytes} \approx 812\text{kB}
 \end{aligned}$$

This 812 kB can be easily placed within the 15 tags requiring only 54 kB per tag – a very feasible requirement.

In practice, one would want to include an extra safety factor in the memory calculation to protect against variability in the number of packets stored in a tag, and not including this factor would assume an  $\infty$ -queue system. However, we believe such a safety factor is already included in the conservative estimate of the storage requirement. This estimate assumes that any copy of the packet in the system at time  $T$  exists in the system for the entire possible lifetime of the packet, but in fact, the shared packets will only exist in the system for some shorter time.

We can also consider trends in the total storage capacity necessary for SWIM by performing the same calculations as above using  $\lambda = \frac{1}{24}$ , varying parameters such as the number of whales. This is shown in Figure 12 using 1 buoy and varying the number of whales from 10 to 40. Storage requirements in SWIM situation grow slightly due to the replication of packets. If the density of whales is larger, then the packet is shared with more whales and more storage is required. This increased storage requirement is, however, mitigated by the fact that the delay is reduced. On the other hand, the expected delay for the non-sharing system is essentially constant (approximately 1175 timesteps) over different numbers of whales, since having more whales in the system offers no advantage. In the non-sharing case, the particular whale carrying the packet must reach the buoy. SWIM shares information, so if there is higher density of whales, there is more sharing. Thus, SWIM achieves smaller delays as the number of whales increases.

As one would expect, the storage requirements also vary considerably for different arrangements of the SWIM stations. If the SWIM stations are placed at the center or near the feeding grounds, packets are offloaded quicker and without excessive sharing of data, resulting in requirements of 19 or 26 kB, respectively, per whale for a system of 15 whales and 1 buoy. The maximum separated buoy case requires

193 kB/whale, and the Poisson distributed case requires 134 kB/whale. Since the time until packet offloading is generally larger in these cases, the whales tend to group and share information more before offloading the packets.

In the multitiered case, the storage requirements may decrease or increase as the mobility of the SWIM stations increases, depending on the speed of the SWIM stations relative to the speed of the whales. For stations moving at speeds significantly higher than the speed of the whales, less memory is required in the whale tags. However, for stations moving at speeds comparable to the whales, significantly more memory is required, for the reasons discussed earlier. For example, a system with 15 whales, and one station moving at speed 9 units/timestep needs 23 kB of memory per whale, while moving at speed 3 units/timestep needs 70 kB/whale, as opposed to the 26 kB/whale required for stationary buoys near the center of feeding grounds.

## 6. CONCLUSIONS

The *Infostations* concept is a networking paradigm that trades delay for improved network capacity. Users are able to experience very fast data rates, at the expense of intermittent network connectivity. The *Infostations* model is just one possible solution in the delay-capacity tradeoff in wireless networks.

We have proposed in this paper to extend the *Infostations* concept by integrating it with the ad hoc networking paradigm. In particular, users may disseminate information packets throughout the system to other users, in hope that one of those replicas will reach one of the collection points when one of these users is within reception distance from a collection station. We refer to this augmented *Infostations* model as the *Shared Wireless Infostation Model* and to the collection stations as SWIM stations.

Clearly, because of the increased redundancy, the delay until a packet reaches one of the collection stations is significantly reduced. The delays of traditional *Infostation* networks are shown to be 1.6 to 3.5 times the delay of the SWIM model. A penalty of transmission bandwidth in the system due to the replicating of information in SWIM is also incurred, in addition to a modest increase in the storage requirements. This additional bandwidth is proportional to the number of duplicate packets in the system.

We have then applied the SWIM concept to the design of an example application system - the biological acquisition system. SWIM stations placed at the surface of the water offer a low-cost solution to recover data collected on whale tags using high bandwidth at the expense of delay in delivery. The sharing of data among the whales, as well as the use of multiple SWIM stations, has been shown to significantly reduce this end-to-end delay.

We first obtained an analytical cumulative distribution function and then, incorporating biological considerations, we used simulations to obtain the relevant values of the parameters for the analytical solution. This allowed us to compute various performance measures, such as the distribution of the packet offload times (useful, for example, in calculation of the Time-To-Live of the packets replicas), and curves of the number of whales infected at the packet offloading time (useful, for example, in designing the tag memory requirements).

Through simulations, several trends and trade-offs were

observed, such as reliability<sup>6</sup> vs. storage space. More specifically, if a packet is stored in the system for a longer period of time and there are more copies, then there is higher probability of the packet eventually reaching one of the offloading stations before it is discarded from the system. Although increasing the number of whales also increases the storage space required, the overall reliability (probability that a packet reaches a collection station) is increased as well. Increasing the number of buoys increases reliability by increasing the cost of the system, though this does not increase the tag memory requirement. Buoy positions and mobility greatly affects the system reliability, while grouping and feeding weights have much smaller impact.

The Shared Wireless Infostation Model extends the *Infostation* paradigm through information sharing between the nodes. This model is well-suited to delay-tolerant applications such as the above biological information acquisition system. Nevertheless, this methodology can be also used to model and evaluate other systems that use the extended *Infostation* model. Though there is no upper bound on the delays incurred in *Infostation* systems, SWIM significantly decreases average delays.

## 7. ACKNOWLEDGMENTS

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<sup>6</sup>the probability that a packet reaches a buoy

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## APPENDIX

### A. FINDING THE OFFLOADING TIME DISTRIBUTION

This first-order differential equation for the number of whales in the infected state at time  $t$  is separable and can be solved with initial condition  $I(0) = 1$  to give the solution

$$I(t) = \frac{N}{1 + e^{-\beta N t}(N - 1)}.$$

Let  $T$  be the time from  $t = 0$  until the information has been first offloaded to any of the buoys. To find its c.d.f., it is first necessary to calculate the probability  $Pr(T \geq t + h)$ , where  $h$  is some small quantity that will go to 0, used to formulate a differential equation. In this case, we assume independent increments; that is,  $Pr(\text{event in } [0, t])$  is independent from  $Pr(\text{event in } [t, t + h])$ .

$$\begin{aligned} Pr(\text{no offload in interval } [t, t + h] \mid \text{no offload by } t) &= 1 - Pr(\text{offload in interval } [t, t + h] \mid \text{no offload by } t) \\ &= 1 - (\text{duration of interval})(\text{rate of offload}) \\ &= 1 - h\gamma I(t) \end{aligned}$$

$$\begin{aligned} \Rightarrow Pr(T > t + h) &= Pr(\text{no offload by } t) * \\ &Pr(\text{no offload in interval } [t, t + h] \mid \text{no offload by } t) \\ &= Pr(T > t)[1 - h\gamma I(t)]. \end{aligned}$$

This can be used to find a differential equation for the cumulative distribution function of  $T$ , which we will call  $F$ . First, the differential equation for  $F(t)$  is found by using the formula for  $Pr(t + h)$  in terms of  $Pr(T > t)$  found above.

$$\begin{aligned} \frac{dF}{dt} &= \lim_{h \rightarrow 0} \frac{F(t+h) - F(t)}{h} \text{ for small } h > 0 \\ &= \lim_{h \rightarrow 0} - \frac{[Pr(T > t+h) - Pr(T > t)]}{h} \\ &= \lim_{h \rightarrow 0} - \frac{1}{h} Pr(T > t)[(1 - h\gamma I(t)) - 1] \\ &= \gamma I(t) Pr(T > t) \\ &= \gamma \frac{N}{1 + e^{-\beta N t}(N - 1)} [1 - F(t)]. \end{aligned}$$

This differential equation is also separable. We solve the differential equation using the initial condition that  $F(0) = \frac{\pi t x_{\text{buoy}}^2 M}{\text{area of grid}}$ , since this is the probability of a whale being placed within range of any of  $M$  SWIM stations at time 0.

**Table 1:  $\chi^2$  stats varying whale and buoy numbers**

Whales	Buoys	Avg $\beta$	Avg $\gamma$	$\chi^2$ Stat
10	1	6.32E-04	8.97E-04	0.052
15	1	6.22E-04	8.98E-04	0.058
20	1	6.10E-04	9.00E-04	0.078
25	1	5.96E-04	8.99E-04	0.058
30	1	5.81E-04	9.01E-04	0.056
35	1	5.66E-04	8.90E-04	0.063
40	1	5.48E-04	9.08E-04	0.065
10	2	6.38E-04	1.82E-03	0.028
15	2	6.22E-04	1.80E-03	0.033
20	2	6.09E-04	1.81E-03	0.037
25	2	5.96E-04	1.81E-03	0.030
30	2	5.80E-04	1.79E-03	0.026
35	2	5.65E-04	1.80E-03	0.038
40	2	5.48E-04	1.80E-03	0.025
10	3	6.33E-04	2.69E-03	0.025
15	3	6.21E-04	2.71E-03	0.029
20	3	6.08E-04	2.73E-03	0.045
25	3	5.95E-04	2.70E-03	0.034
30	3	5.79E-04	2.70E-03	0.036
35	3	5.66E-04	2.71E-03	0.034
40	3	5.48E-04	2.70E-03	0.032
10	4	6.30E-04	3.63E-03	0.017
15	4	6.20E-04	3.59E-03	0.026
20	4	6.10E-04	3.59E-03	0.028
25	4	5.95E-04	3.58E-03	0.031
30	4	5.79E-04	3.63E-03	0.035
35	4	5.64E-04	3.62E-03	0.041
40	4	5.48E-04	3.57E-03	0.027

Then,

$$F(t) = 1 - K \left( \frac{N - 1}{e^{\beta N t} + N - 1} \right)^{\frac{\gamma}{\beta}}$$

is the c.d.f. for  $T$  with  $K = \left[ \frac{N-1}{N} \right]^{-\frac{\gamma}{\beta}} \left[ 1 - \frac{\pi t x_{\text{buoy}}^2 M}{\text{area of grid}} \right]$ . Notice that the property  $\lim_{t \rightarrow \infty} F(t) = 1$ , a necessary condition of any c.d.f., is satisfied.

## B. ANALYSIS VS. SIMULATION $\chi^2$ TEST

Table 1 compiles data using the random midway mobility model, an area of size 300 by 300, the transmission/reception radius for the whales of 7.5 units, transmission/reception for the buoys of 15 units, showing good agreement between analytical solution and simulation.